Exponential Functions

There is a connection between population growth, radioactive decay, musical scales, and compound interest. They seem to have little in common, but you can model any of them using an exponential function.

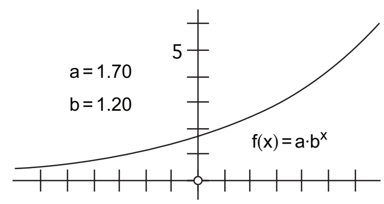
An exponential function has the general form $f(x) = ab^x$, where $a \neq 0$, $b > 0$, and $b \neq 1$.

PROPERTIES OF THE GRAPH

Before using an exponential function to model a real-world problem, take some time to familiarize yourself with the graph.

To create a parameter, choose **Graph | New Parameter.**

1. In a new sketch, create parameters a and b.
2. Graph the function $f(x) = a \cdot b^x$ by choosing **Graph | Plot New Function.** Click the parameters in the sketch to enter them into the function.



To change the value of a parameter, either double-click it or select it and press the + or − key. (To change the size of the steps for the + or − keys, select the parameter and choose **Edit | Properties | Parameter.** Change the Keyboard Adjustments value to 0.1 unit.)

3. The graph is plotted on the screen. Change the values of the parameters, and observe the resulting changes in the graph. Try several values for each parameter.

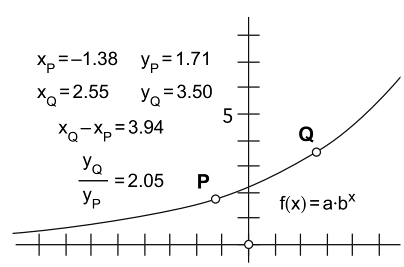
Q1 What are the x- and y-intercepts of the graph? Explain how the intercepts are related to parameters a and b.

Q2 The value of $f(x)$ tends to get close to zero either on the left side or on the right. What parameter values determine which side it is?

Q3 In the general form of the exponential function, there are three constraints ($a \neq 0$, $b > 0$, and $b \neq 1$). Explain the reason for each of these constraints.

Next you'll investigate how the function behaves by comparing the coordinates of two points on the graph.

4. Change the parameters so that $a = 2.00$ and $b = 1.30$.
5. Construct two points on the function graph. Label them P and Q.
6. Select both points and choose **Measure | Abscissa (x).** Select the points again and choose **Measure | Ordinate (y).**
7. Calculate the values $x_Q - x_P$ and y_Q / y_P.



You can change the scale of the axes to give you more precise control over the positions of the points.

Q4 Drag point Q one unit to the right of P, so that the difference $x_Q - x_P$ is as close to 1.00 as you can make it. What is the value of the ratio y_Q / y_P? Drag point P to a different

position on the graph, and again drag Q so it's one unit to the right. What is the value of the ratio y_Q/y_P? Why do you get this result?

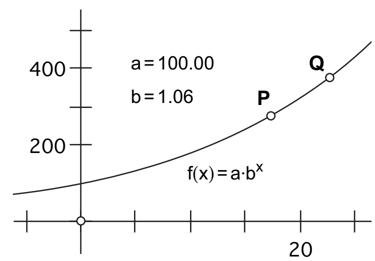
Q5 Now use a difference other than 1. Drag the points so that $x_Q - x_P$ is approximately 2.5. What is the ratio? Drag them to another position, but with the x difference still equal to 2.5. What is the ratio now? Explain.

DOUBLING PERIOD AND HALF-LIFE

Exponential functions can be used to solve a number of real-life problems. First change the function so that it shows the value of $100 invested at an effective annual yield of 6%.

Effective annual yield is not the same thing as interest rate. That's another topic.

8. Edit parameters a and b so that the function has the definition $f(x) = 100(1.06)^x$. This shows the value of $100 invested at an effective annual yield of 6%. (The x variable is in years, and y is in dollars.)



Setting the grid to rectangular allows you to adjust each axis independently of the other.

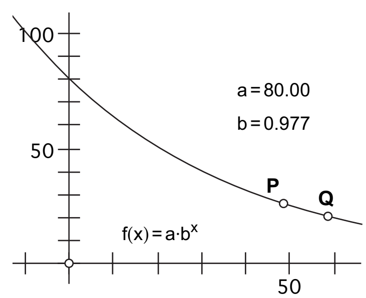
9. At first you can't see the graph because the y-axis doesn't go up to 100. To adjust the axes, choose **Graph | Grid Form | Rectangular Grid.** Then drag tick mark numbers on each axis so that you can see the results for the first 25 years.

To see more decimal places in a parameter, select the parameter and choose **Edit | Properties | Value.** Change the Precision setting.

Q6 How long will it take to double your money? Drag the points so that the ratio is 2.00. What is the difference in their x-coordinates? This number is called the *doubling period.*

An exponential function can also be used to model the decay of radioactive cesium.

10. To model the decay of 80 g of cesium, change the function definition to $f(x) = 80(0.977)^x$. Adjust the axes appropriately. The value of x is still in years, but the value of y is in grams.



Q7 If you start with 80 g, you will have less cesium every year. How long would it take to lose half of it? Explain how you found the answer. This number is called the *half-life* of cesium.

Q8 Although cesium decays, as opposed to growing, you can still calculate its doubling period. Drag the two points until you find a position where the ratio is 2.00. What is the difference in the x-coordinates? Explain how this verifies your answer to Q7.